

Operator limits of random matrices,

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By and large, the study of random matrices is an asymptotic spectral theory. For a given ensemble of n by n matrices, one aims to prove limit theorems for the eigenvalues as the dimension tends to infinity. One of the more remarkable aspects of the subject is that it has introduced important new points of concentration in the space of distributions. Take for example the Tracy-Widom laws. First discovered as the fluctuation limit for the spectral radius of certain Gaussian Hermitian matrices, these laws are now understood to govern the behavior of a wide range of nonlinear phenomena in mathematical physics (exclusion processes, random growth models, etc.)

My aim here will be to describe a relatively new approach to limit theorems for random matrices. Instead of focussing on some particular spectral statistic, one rather understands the large dimensional limit as a continuum limit, demonstrating that the matrices themselves converge to some random differential operators. This method is especially suited to the beta ensembles of random matrix theory. The latter generalize the classical Gaussian Unitary and Orthogonal Ensembles, and can be viewed in their own right as models of coulomb gases.

In the first lecture I will review these just mentioned classical ensembles and their underlying analytic structure which paved the way for Tracy and Widom's original work. After that I'll introduce the beta ensembles and our main players, the stochastic Airy and Bessel operators, which characterize a generalized family of Tracy-Widom laws. Lecture two will focus on the proofs of the underlying operator convergence which define the stochastic Airy and Bessel operators. The last lecture will be devoted to upshots and applications of these random operators: tail estimates for general beta Tracy-Widom, the Baik-Ben Arous-Peche phase transition, and universality.

